2.4 Multiplying Mixed Numbers

Essential Question How do you multiply a mixed number by a fraction?
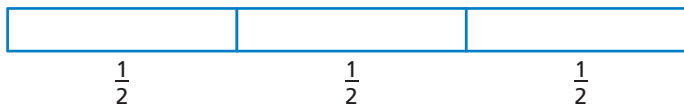
1 ACTIVITY: Multiplying a Mixed Number and a Fraction

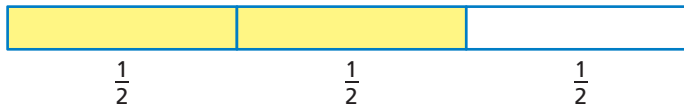**Work with a partner. Use a diagram to find the product.**

a. **Sample:** $1\frac{1}{2} \times \frac{2}{3}$

What is two-thirds of $1\frac{1}{2}$?

> Think of $1\frac{1}{2}$ as three halves.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

> Two of the halves is 1.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

So, $1\frac{1}{2} \times \frac{2}{3} = 1$.

b. $\frac{4}{9} \times 2\frac{1}{4}$

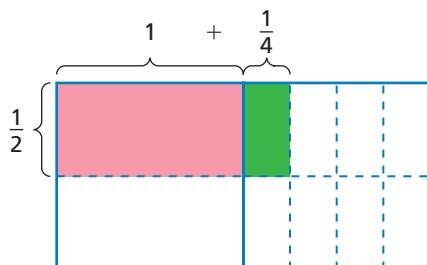c. $2\frac{1}{4} \times \frac{1}{2}$

d. $\frac{1}{3} \times 3\frac{3}{4}$

2 ACTIVITY: Multiplying a Mixed Number and a Fraction

Work with a partner. How many square feet are in the piece of fabric?
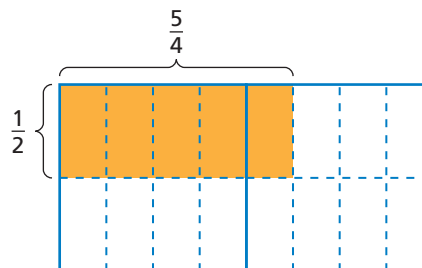
$1\frac{1}{4}$ ft

$\frac{1}{2}$ ft

a. Use the Distributive Property and find the sum of the two pieces.

1 $+$ $\frac{1}{4}$

$\frac{1}{2}$

b. Rewrite the mixed number as an improper fraction and multiply.

$\frac{5}{4}$

$\frac{1}{2}$

ACTIVITY: Buried Treasure Game

Number of Players: 2

Taking turns, each player will

- choose a treasure location.
- roll a number cube as many times as there are blanks in the expression.
- place the numbers in the blanks to form the largest possible value for that treasure location.
- check each other's work.

Players then total the values of their treasures (sum of the three expressions). The player with the larger total wins the game.
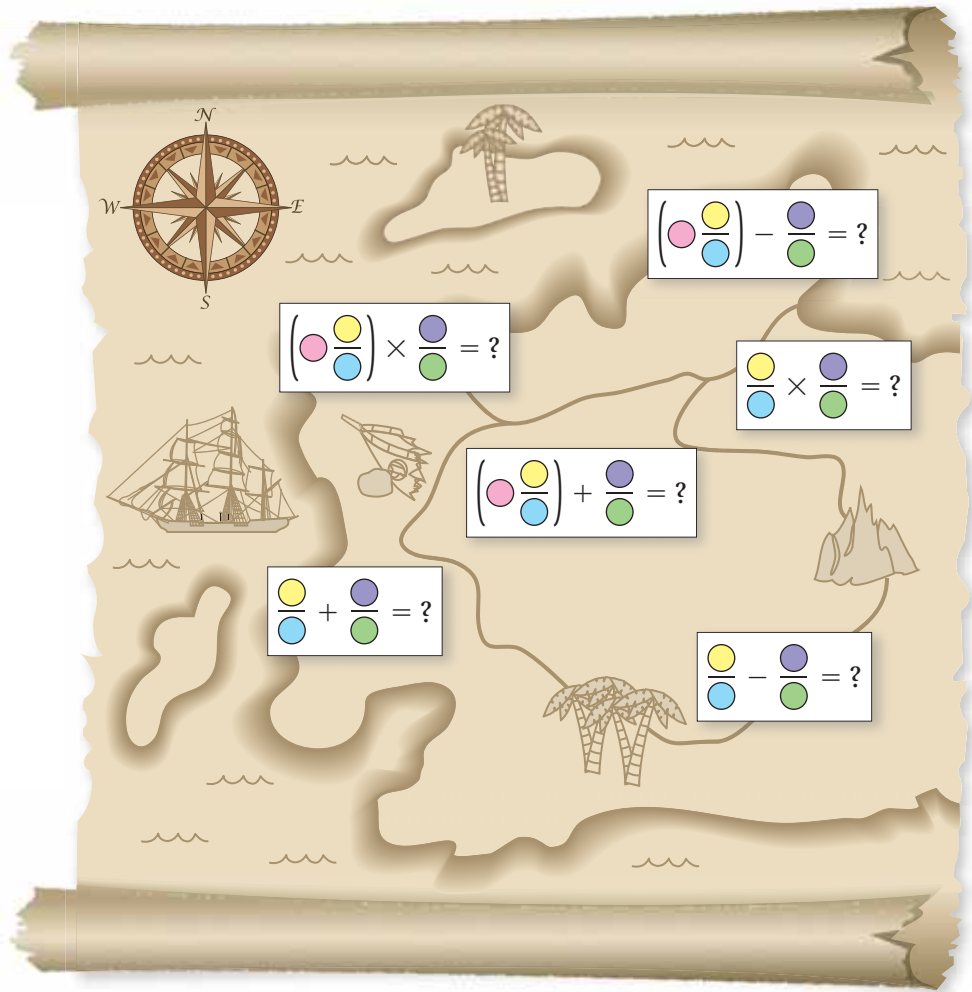
What Is Your Answer?

4. IN YOUR OWN WORDS How do you multiply a mixed number by a fraction?

Practice Use what you learned about multiplying mixed numbers to complete Exercises 8–15 on page 66.

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Lesson Tutorials
BigIdeasMath.com

EXAMPLE 1 Using the Distributive Property

Use the Distributive Property to find $\frac{1}{2} \times 2\frac{3}{4}$.

$$\frac{1}{2} \times 2\frac{3}{4} = \frac{1}{2} \times \left(2 + \frac{3}{4}\right)$$ Rewrite $2\frac{3}{4}$ as the sum $2 + \frac{3}{4}$.

$$= \left(\frac{1}{2} \times 2\right) + \left(\frac{1}{2} \times \frac{3}{4}\right)$$ Use the Distributive Property.

$$= 1 + \frac{3}{8}$$ Multiply.

$$= 1\frac{3}{8}$$ Add.

On Your Own

Now You're Ready
Exercises 4–7

Use the Distributive Property to find the product.

1. $\frac{2}{3} \times 1\frac{1}{2}$ **2.** $\frac{3}{5} \times 2\frac{1}{6}$ **3.** $\frac{1}{4} \times 4\frac{1}{5}$ **4.** $\frac{2}{7} \times 3\frac{3}{4}$

Key Idea

Multiplying Mixed Numbers
Write each mixed number as an improper fraction. Then multiply as you would with fractions.

EXAMPLE 2 Multiplying a Fraction and a Mixed Number

Find $\frac{1}{2} \times 2\frac{3}{4}$.   Estimate $\frac{1}{2} \times 3 = 1\frac{1}{2}$

$$\frac{1}{2} \times 2\frac{3}{4} = \frac{1}{2} \times \frac{11}{4}$$ Write $2\frac{3}{4}$ as the improper fraction $\frac{11}{4}$.

$$= \frac{1 \times 11}{2 \times 4}$$ Multiply the numerators and the denominators.

$$= \frac{11}{8}, \text{ or } 1\frac{3}{8}$$ Simplify.

∴ So, the product is $1\frac{3}{8}$.   **Reasonable?** $1\frac{3}{8} \approx 1\frac{1}{2}$ ✓

EXAMPLE **3**

Multiplying Mixed Numbers

Find $1\dfrac{4}{5} \times 3\dfrac{2}{3}$.   **Estimate** $2 \times 4 = 8$

$$1\dfrac{4}{5} \times 3\dfrac{2}{3} = \dfrac{9}{5} \times \dfrac{11}{3}$$ Write $1\dfrac{4}{5}$ and $3\dfrac{2}{3}$ as improper fractions.

$$= \dfrac{\overset{3}{\cancel{9}} \times 11}{5 \times \underset{1}{\cancel{3}}}$$ Multiply fractions. Divide out the common factor 3.

$$= \dfrac{33}{5}, \text{ or } 6\dfrac{3}{5}$$ Simplify.

So, the product is $6\dfrac{3}{5}$.   **Reasonable?** $6\dfrac{3}{5} \approx 8$ ✓

On Your Own

Now You're Ready
Exercises 8–23

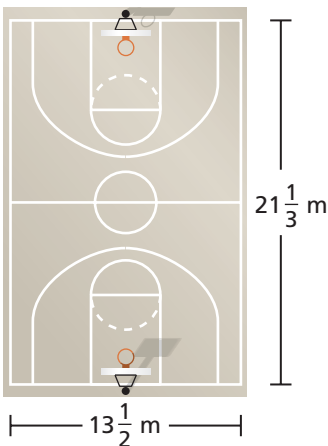**Multiply. Write the answer in simplest form.**

5. $\dfrac{1}{3} \times 1\dfrac{1}{6}$ **6.** $3\dfrac{1}{2} \times \dfrac{4}{9}$ **7.** $1\dfrac{7}{8} \times 2\dfrac{2}{5}$ **8.** $5\dfrac{5}{7} \times 2\dfrac{1}{10}$

EXAMPLE **4** **Real-Life Application**

A city is resurfacing a basketball court. Find the area of the court.

Estimate $21 \times 14 = 294$

$21\dfrac{1}{3}$ m

$13\dfrac{1}{2}$ m

$$A = \ell w$$ Write the formula for the area of a rectangle.

$$= 21\dfrac{1}{3} \times 13\dfrac{1}{2}$$ Substitute $21\dfrac{1}{3}$ for ℓ and $13\dfrac{1}{2}$ for w.

$$= \dfrac{64}{3} \times \dfrac{27}{2}$$ Write $21\dfrac{1}{3}$ and $13\dfrac{1}{2}$ as improper fractions.

$$= \dfrac{\overset{32}{\cancel{64}} \times \overset{9}{\cancel{27}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}}}$$ Multiply fractions. Divide out common factors.

$$= 288$$ Simplify.

The area of the court is 288 square meters.

Reasonable? $288 \approx 294$ ✓

On Your Own

9. Find the area of a rectangular air hockey table that is $8\dfrac{1}{4}$ feet by $4\dfrac{3}{8}$ feet.

✓ Vocabulary and Concept Check

1. **VOCABULARY** What is an improper fraction?

2. **WRITING** Describe how to multiply two mixed numbers.

3. **OPEN-ENDED** Write two mixed numbers between 3 and 4 that have a product between 9 and 12.

Practice and Problem Solving

Use the Distributive Property to find the product.

4. $\dfrac{1}{4} \times 2\dfrac{2}{7}$　　　**5.** $\dfrac{5}{6} \times 2\dfrac{2}{5}$　　　**6.** $\dfrac{5}{9} \times 4\dfrac{1}{2}$　　　**7.** $\dfrac{2}{15} \times 5\dfrac{5}{8}$

Multiply. Write the answer in simplest form.

8. $1\dfrac{1}{3} \times \dfrac{2}{3}$　　　**9.** $6\dfrac{2}{3} \times \dfrac{3}{10}$　　　**10.** $2\dfrac{1}{2} \times \dfrac{4}{5}$　　　**11.** $\dfrac{3}{5} \times 3\dfrac{1}{3}$

12. $7\dfrac{1}{2} \times \dfrac{2}{3}$　　　**13.** $\dfrac{5}{9} \times 3\dfrac{3}{5}$　　　**14.** $\dfrac{3}{4} \times 1\dfrac{1}{3}$　　　**15.** $3\dfrac{3}{4} \times \dfrac{2}{5}$

16. $4\dfrac{3}{8} \times \dfrac{4}{5}$　　　**17.** $\dfrac{3}{7} \times 2\dfrac{5}{6}$　　　**18.** $1\dfrac{3}{10} \times 18$　　　**19.** $15 \times 2\dfrac{4}{9}$

20. $1\dfrac{1}{6} \times 6\dfrac{3}{4}$　　　**21.** $2\dfrac{5}{12} \times 2\dfrac{2}{3}$　　　**22.** $5\dfrac{5}{7} \times 3\dfrac{1}{8}$　　　**23.** $2\dfrac{4}{5} \times 4\dfrac{1}{16}$

**ERROR ANALYSIS** Describe and correct the error in finding the product.

24.
✗ $4 \times 3\dfrac{7}{10} = 12\dfrac{7}{10}$

25.
✗ $2\dfrac{1}{2} \times 7\dfrac{4}{5} = (2 \times 7) + \left(\dfrac{1}{2} \times \dfrac{4}{5}\right)$
$= 14 + \dfrac{2}{5} = 14\dfrac{2}{5}$

26. **VITAMIN C** A vitamin C tablet contains $\dfrac{1}{40}$ gram of vitamin C. You take $1\dfrac{1}{2}$ tablets every day. How many grams of vitamin C do you take every day?

27. **SCHOOL BANNER** A banner is made for a football rally.

　　a. What is the area of the banner?

　　b. A $\dfrac{1}{4}$-foot border is added on each side. What is the new area of the banner?

GO PANTHERS!　$1\dfrac{1}{2}$ ft

$4\dfrac{2}{3}$ ft
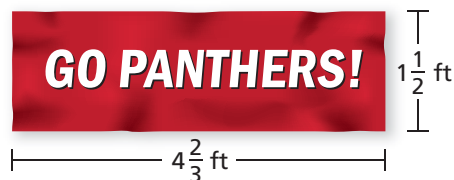
ALGEBRA Evaluate the expression when $x = 5\frac{5}{8}$, $y = 2\frac{4}{9}$, and $z = 1\frac{5}{16}$.

28. $5\frac{1}{7} \cdot z$

29. xy

30. $x \cdot 2\frac{2}{9} - 6\frac{3}{7}$

31. $yz + 5\frac{1}{3}$

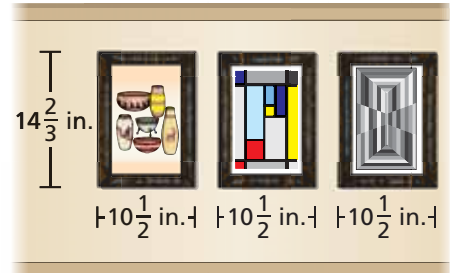**Multiply. Write the answer in simplest form.**

32. $\frac{4}{7} \times 4\frac{3}{8} \times \frac{5}{6}$

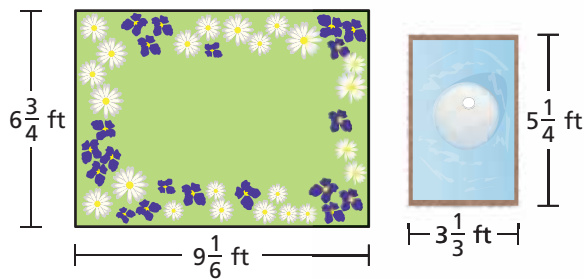**33.** $3\frac{3}{4} \times 5\frac{1}{6} \times 8$

34. $2\frac{1}{10} \times \frac{5}{9} \times 4\frac{2}{3}$

35. $1\frac{1}{15} \times 5\frac{2}{5} \times 4\frac{7}{12}$

36. PICTURES Three pictures hang side by side on a wall. What is the total area of the wall covered by the pictures?

$14\frac{2}{3}$ in.

$\vdash 10\frac{1}{2}$ in.$\dashv$ $\vdash 10\frac{1}{2}$ in.$\dashv$ $\vdash 10\frac{1}{2}$ in.$\dashv$

37. REASONING Is the product of two positive mixed numbers ever less than 1? Explain.

38. GARDEN You plan to add a fountain to your garden.

$6\frac{3}{4}$ ft

$9\frac{1}{6}$ ft

$5\frac{1}{4}$ ft

$\vdash 3\frac{1}{3}$ ft $\dashv$

 a. Draw a diagram of the fountain in the garden. Label the dimensions.

 b. Describe 2 methods for finding the area of the garden that surrounds the fountain.

 c. Find the area. Which method did you use, and why?

39. Reasoning The cooking time for a ham is $\frac{2}{5}$ hour for each pound.

 a. How long should you cook a ham that weighs $12\frac{3}{4}$ pounds?

 b. Dinner time is 4:45 P.M. What time should you start cooking the ham?

Fair Game Review What you learned in previous grades & lessons

Multiply. Write the answer in simplest form.

40. $\frac{2}{5} \times \frac{3}{7}$

41. $\frac{1}{4} \times \frac{2}{3}$

42. $\frac{5}{6} \times \frac{8}{15}$

43. $\frac{7}{8} \times \frac{2}{9}$

Use the Distributive Property to rewrite the expression.

44. $5(x + 6)$

45. $9(x - 3)$

46. $4(7 + x)$

47. $12(x - 8)$

48. MULTIPLE CHOICE How many inches are in 5 yards?

Ⓐ 15   Ⓑ 60   Ⓒ 120   Ⓓ 180